Exercise 20

Find dy/dx by implicit differentiation.

$$\tan(x-y) = \frac{y}{1+x^2}$$

Solution

Differentiate both sides with respect to x.

$$\frac{d}{dx}[\tan(x-y)] = \frac{d}{dx}\left(\frac{y}{1+x^2}\right)$$

$$[\sec^2(x-y)]\cdot\frac{d}{dx}(x-y) = \frac{\left[\frac{d}{dx}(y)\right](1+x^2) - \left[\frac{d}{dx}(1+x^2)\right]y}{(1+x^2)^2}$$

$$[\sec^2(x-y)]\cdot(1-y') = \frac{(y')(1+x^2)-(2x)y}{(1+x^2)^2}$$

$$(1-y')\sec^2(x-y) = \frac{(1+x^2)y' - 2xy}{(1+x^2)^2}$$

Solve for y'.

$$\sec^2(x-y) - y'\sec^2(x-y) = \frac{y'}{1+x^2} - \frac{2xy}{(1+x^2)^2}$$

$$\sec^2(x-y) + \frac{2xy}{(1+x^2)^2} = \left[\frac{1}{1+x^2} + \sec^2(x-y)\right]y'$$

$$y' = \frac{\sec^2(x-y) + \frac{2xy}{(1+x^2)^2}}{\frac{1}{1+x^2} + \sec^2(x-y)}$$

$$y' = \frac{(1+x^2)^2\sec^2(x-y) + 2xy}{1+x^2+(1+x^2)^2\sec^2(x-y)}$$